THE UNIVERSITY *of York*

Discussion Papers in Economics

No. 2008/06

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Multitasking, quality and pay for performance

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April 10, 2008

Abstract

We present a model of optimal contracting between a purchaser and a provider of health services when quality has two dimensions. We assume that one dimension of quality is contractible (dimension 1) and one dimension is not contractible (dimension 2). We show that the optimal incentive scheme for the contractible dimension depends critically on the extent to which quality 1 increases or decreases the marginal cost and marginal benefit of quality 2 (i.e. substitutability or complementarity). If the two quality dimensions are substitutes, three possible solutions arise: a) the optimal incentive scheme is *high powered*: the incentive is equal to the marginal benefit of quality dimension 1 and the optimal quality in dimension 2 is zero; b) the optimal incentive scheme is *low powered*: both quality dimensions are positive; the incentive is below the marginal benefit of quality dimension 1; c) it is not optimal to introduce pay for performance as the gain of welfare from an increase in quality dimension 1 is lower than the loss of welfare from an increase in quality dimension 2. If the two quality dimensions are complements the incentive scheme is always *high powered*: the incentive is above the marginal benefit of dimension 1 and both quality dimensions are positive.

Keywords: quality, altruism, incentives. JEL: D82; I11; I18; L51

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1 Introduction

Policymakers aim to design incentive schemes that encourage better performance in the health care sector. This is often referred to as *Paying for Performance*. For example, the Medicare Programme in the U.S. provides higher transfers to hospitals that perform well according to measurable quality indicators, such as rates of cervical cancer screening and hemoglobin testing for diabetic patients (Rosenthal et al., 2005). In the United Kingdom general practitioners who perform well on certain quality indicators, such as the measurement of blood pressure and cholesterol in patients with ischemic heart disease, can receive substantial financial rewards. These can amount to about 20% of a general-practitioner's budget (Doran et al., 2006). PROMINA Health System, an Atlanta-area federation of eight hospitals and nearly 4000 employed and outside physicians, has agreed on sizable quality reimbursement incentives for about 1500 physicians in affiliated practices (AIS, 2003). If a practice meets a certain level of compliance with quality standards (e.g. that a given percentage of pneumonia patients must receive an antibiotic within four hours of being admitted) they receive reimbursement for all services to CIGNA/PROMINA patients that is set a 5% higher multiple of Medicare reimbursement than the baseline multiple such as specified in PROMINA's contract with CIGNA HealthCare of Georgia.

The Pay for Performance (P4P) programs outlined above define quality in such a way that it is verifiable. That is, the reimbursement contract between the payer and the provider must be written such that quality indicators can be observed and verified ex post by a third party (court). However, a major issue in rewarding performance is that while some quality dimensions are contractible through performances indicators, other dimensions of quality are not. For example, both communication about medical conditions, and hemoglobin testing affect the quality of care for diabetic patients. While the latter dimension can be verified by a third party, the former dimension is not. As is well known from the contract literature, problems of non-verifiability and multi-tasking may impose severe difficulties in effective incentive design (Holmstrom and Milgrom, 1991; Baker 1992).

Recently, Eggleston (2005) provides a model with two quality dimensions. She shows that if one dimension of quality is contractible, while one dimension of quality is not, then the introduction of an P4P-program may increase service on the verifiable quality dimension but may decrease service on the non-verifiable one. She argues that incentives for non-verifiable quality can be restored by reducing P4P on verifiable quality.

While Eggelston's argument seems intuitive, reducing P4P-incentives comes at a cost since it reduces service on the verifiable quality. This reduces patients' benefit of treatment, and the purchaser's utility. There is thus an issue of how the sponsor (payer) should adjust the reimbursement contract to ensure that her objectives are fulfilled. That is, what is the optimal strength of the P4P-incentives for the observed quality and under which conditions is the optimal incentive positive. The purpose of this paper is to investigate such conditions.

We show that the optimal incentive scheme depends critically on the extent to which quality 1 increases or decreases the marginal cost and marginal benefit of quality 2 (i.e. the extent to which quality 1 and 2 are substitutes or complements). If the two quality dimensions are *substitutes*, we show that three possible solutions can arise. a) The optimal incentive scheme is *high powered*: the P4P-incentive scheme is equal to the marginal benefit of quality dimension 1 and the optimal quality in dimension 2 is zero. This result arises when the quality dimension that is not contractible falls quickly to the minimum when the price is raised, while the benefits from the quality dimension that is contractible are large. b) The optimal incentive scheme is *low powered*: the P4P-incentive scheme is below the marginal benefit of quality dimension 1. Both quality dimensions are positive. This result arises when the benefits from the quality dimension that is contractible are relatively more important but need to be traded off with the reductions in the quality dimension that is not contractible. c) The incentive scheme *breaks down*: it is not optimal to introduce pay for performance as the gain of welfare from an increase in quality dimension 1 is lower than the loss of welfare from an increase in quality dimension 2. This result arises when the benefits from the quality dimension that is not contractible are relatively more important.

If the two quality dimensions are *complements*, the incentive scheme is always *high* *powered.* The P4P-incentive scheme is above the marginal benefit of quality dimension 1. Both quality dimensions are positive.

Like Eggleston (2005) we also compare our solutions with what can be obtained if both dimensions of quality are verifiable. Obviously, the second best quality, when quality dimension 2 is not contractible, is generally different from the first best quality, when quality dimension 2 is also contractible, however not necessarily lower. Second best verifiable quality might be highest in second best if the two dimensions of quality are complements in the providers costs of producing quality.

This study contributes to the literature on provider incentives in health care. Despite the increase in the use of performance indicators, most of the existing theoretical literature assumes that quality is not contractible (for example Pope, 1989; Ma, 1994; Rogerson, 1994; Ellis, 1998; Ellis and McGuire, 1990; Chalkley and Malcomson, 1998a and 1998b; Mougeot and Naegelen, 2005). As quality indicators become increasingly available, quality becomes partially contractible. Therefore, there is increasing scope for analysing incentive schemes within this imperfect environment.

The paper is organised as follows. Section 2 introduces the main assumptions of the model and derives the equilibrium price when only one dimension of quality is contractible. Sections 3 derives the first best solution, when both dimensions of quality are contractible, and compares it with the second best solution derived in section 2. Section 4 provides comparative statics with respect to the price. Section 5 concludes.

2 The model

There are two active players, the sponsor (the payer or a purchaser of health services) and the provider (hospital, family doctor). The sponsor provides reimbursement to the provider, and the provider exerts effort on two quality tasks. In addition, fully insured patients, whose benefit is increasing in the quality provided on both tasks, seek treatment. We further assume that the provider is not demand constrained.

The model is solved by backwards induction, starting with the provider's choice of quality levels.

2.1 The provider

There are two dimensions of quality, q_1 and q_2. The disutility in monetary terms from exerting quality effort q_1, q_2 is $C(q_1, q_2)$. The disutility is increasing in quality and strictly convex: $C_{q_i} > 0$, and $C_{q_i q_i} > 0$, where $C_{q_i} := \partial C_i / \partial q_i$ and $C_{q_i} := \partial^2 C_i / \partial q_i^2$ for $i = 1, 2$. If the two dimensions of quality are substitutes, then an increase in quality 1 increases the marginal disutility of quality 2 and $C_{q_2 q_1} > 0$. If they are complements, an increase in quality 1 reduces the marginal disutility of quality 2 and $C_{q_2 q_1} < 0$. We also assume $C_{q_1}(0, q_2) = C_{q_2}(q_1, 0) = 0$.

Patients' benefit from receiving quality q_1 and q_2 is $B(q_1, q_2)$ with $B_{q_i} > 0$, and $B_{q_i q_i} < 0$, $i = 1, 2$. If $B_{q_1 q_2} = 0$ then the two dimensions of quality are independent. If $B_{q_1 q_2} < 0$ then an increase in quality 1 decreases the marginal benefit of quality 2 (the two dimensions of quality are substitutes). If $B_{q_1 q_2} > 0$ then an increase in quality 1 increases the marginal benefit of quality 2 (the two dimensions of quality are complements). We will consider all these three cases.

To simplify the exposition and without loss of generality, we assume that the third-order derivatives on patients' benefit and provider's cost are zero.

The incentive scheme is based only on the verifiable dimension of quality q_1. That is, we assume that no contract on q_2 can be enforced: it is prohibitively costly to specify this outcome ex ante in such a way that it can be verified by a court ex post. Therefore, the payment can be based only on q_1 and not q_2. The payment for each unit of observed quality q_1 is $p \geq 0$. The provider also receives a lump-sum payment $T \geq 0$.

The provider is semi altruistic. Altruism is captured by the parameter $\alpha \geq 0$. Provider's utility from providing quality q_1 and q_2 to a representative patient is

$$U = T + pq_1 + \alpha B(q_1, q_2) - C(q_1, q_2) \tag{1}$$

subject to $q_1 \geq 0$, $q_2 \geq 0$. We will show below that in equilibrium the non-negativity constraint for quality 1 is never binding ($q_1 > 0$) while for quality 2 it might be binding. Suppose that quality 2 is also positive in equilibrium ($q_2 > 0$). Then the optimal level of quality provided by the provider are given by the following First Order Conditions (FOCs):

$$p + \alpha B_{q_1}(q_1, q_2) = C_{q_1}(q_1, q_2) \tag{2}$$

$$\alpha B_{q_2}(q_1, q_2) = C_{q_2}(q_1, q_2) \tag{3}$$

The optimal quality for dimension 1 is determined such that the marginal benefit from the price plus the altruistic component are equal to the marginal disutility of providing quality. The optimal quality for dimension 2 is determined such that the marginal benefit from the altruistic component is equal to the marginal disutility.

In the Appendix we show that when the Second Order Conditions (SOCs) are satisfied then $U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2 > 0$ and

$$\frac{\partial q_1}{\partial p} = \frac{-\alpha B_{q_2q_2} + C_{q_2q_2}}{U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2} > 0, \quad \frac{\partial q_2}{\partial p} = \frac{\alpha B_{q_1q_2} - C_{q_1q_2}}{U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2} \gtrless 0. \tag{4}$$

A higher price always increases quality dimension 1 whenever the SOC condition is satisfied ($\frac{\partial q_1}{\partial p} > 0$). This follows since a higher price increases the provider's marginal benefit of providing the verifiable quality. He therefore responds by increasing q_1.

The effect of an increased price on the non-verifiable quality q_2 depends on whether the two quality dimensions are substitutes, independent or complements in patients' benefits and provider's cost.[1]

From equation (4) it follows that an increase in price decreases quality dimension 2 when the two quality dimensions are substitutes in patient's benefit and in provider's cost. If the patient's benefit and provider's cost function is separable in the two quality dimensions then a higher price has no effect on quality 2 but still increases quality 1. A

[1] The two quality dimensions are *substitutes* in patient's benefit and in provider's cost if $C_{q_1q_2} > 0$ and $B_{q_1q_2} < 0$, or if $(\alpha B_{q_1q_2} - C_{q_1q_2}) < 0$. The two quality dimensions are independent if the patient's benefit and provider's disutility function is separable in the two quality dimensions ($B_{q_1q_2} = C_{q_1q_2} = 0$). The two quality dimensions are *complements* in patient's benefit and in provider's cost when $C_{q_1q_2} < 0$ and $B_{q_1q_2} > 0$, or if $(\alpha B_{q_1q_2} - C_{q_1q_2}) > 0$.

higher price increases quality 2 if the two quality dimensions are complements in patient's benefit and in provider's cost. In this case introducing a positive price is clearly welfare improving for the patients (compared to no price) although there is still an issue of how to set the optimal price.

Finally if the constraint $q_2 \geq 0$ is binding with strict equality (which arises when $\alpha B_{q_2} - C_{q_2} < 0$), then the FOC for quality 1 is:

$$p + \alpha B_{q_1}(q_1, q_2 = 0) = C_{q_1}(q_1, q_2 = 0) \tag{5}$$

and $\partial q_1 / \partial p = 1/(-U_{q_1 q_1}) > 0$.

2.2 The purchaser

The purchaser maximises the difference between patients' benefit and the transfers to the provider $B(q_1, q_2) - T - pq_1$ subject to the participation constraint: $U \geq 0$ or $T + pq_1 \geq C(q_1, q_2) - \alpha B(q_1, q_2)$. Since this is binding with equality, the problem becomes:[2]

$$\max_{p \geq 0} \; W = (1+\alpha) B(q_1(p), q_2(p)) - C(q_1(p), q_2(p)) \tag{6}$$

subject to:

$$p + \alpha B_{q_1}(q_1, q_2) - C_{q_1}(q_1, q_2) \leq 0, \quad q_1 \geq 0 \tag{7}$$

$$\alpha B_{q_2}(q_1, q_2) - C_{q_2}(q_1, q_2) \leq 0, \quad q_2 \geq 0, \tag{8}$$

where the inequalities in the incentive–compatibility constraints hold with complementary slackness. The question is: will a strictly positive price increase the purchaser's utility?

[2] We could assume instead that the purchaser maximises a utilitarian welfare function. Define λ as the opportunity cost of public funds. Then a utilitarian welfare function is given by $B - (1+\lambda)(T + pq_1) + U$, which after substituting for $U = 0$ and setting $T + pq_1 = C - \alpha B$, provides $B(1 + \alpha + \lambda\alpha) - (1+\lambda) C$. This formulation is similar to Boadway, Marchand and Sato (2004). Chalkley and Malcomson (1998b) argue that this formulation leads to double counting of the altruistic component, and that the altruistic component into the welfare function should be excluded. If this approach is followed instead, then the welfare function is instead: $B(1 + \lambda\alpha) - (1+\lambda) C$. These alternative formulations would not qualitative impact on our main results.

The trade-off is that a higher price increases quality in dimension 1 and therefore welfare, but might also reduce quality in dimension 2, which reduces welfare.

The FOC with respect to price, if an interior solution exists ($q_2 \geq 0$ is not binding with strict equality), is:[3]

$$\frac{dW(q_1(p), q_2(p))}{dp} = [(1+\alpha)B_{q_1} - C_{q_1}](\partial q_1/\partial p) + [(1+\alpha)B_{q_2} - C_{q_2}](\partial q_2/\partial p) = 0 \quad (9)$$

By using the provider's FOCs ($\alpha B_{q_1} - C_{q_1} = -p$), the optimal price is given by

$$p^* = B_{q_1} + B_{q_2}\frac{\partial q_2/\partial p}{\partial q_1/\partial p} \quad (10)$$

The optimal price is set equal to the marginal benefit of quality 1 adjusted for the the ratio of the responsiveness of the two quality dimensions to price times the marginal benefit of quality 2. From this it follows that the optimal price will be above, equal or below the marginal benefit of quality 1 depending on whether the two quality dimensions are substitutes, independent or complements in patients' benefits and provider's cost. If the two dimensions are substitutes then the optimal price is below the marginal benefit of quality 1: $p^* < B_{q_1}(q_1(p^*), q_2(p^*))$. If a higher price has no effect on quality 2, then the price is equal to the marginal benefit of quality 1: $p^* = B_{q_1}(q_1(p^*), q_2(p^*))$. Finally, if the two dimensions are complements, then the price is set above the marginal benefit of quality 1, $p^* > B_{q_1}(q_1(p^*), q_2(p^*))$.

If the optimal price is above the marginal benefit of quality 1, $p^* \geq B_{q_1}$, we call the incentives *high-powered.* Similarly, if $p^* < B_{q_1}$, then the incentives are *low-powered.*

Notice that if the two dimensions are substitutes, then there is always a level of $p = \overline{p}$ such that the level of quality 2 is zero. Analytically, if $\partial q_2/\partial p < 0$ then $\exists p \geq \overline{p}$ such that $q_2 = 0 \ \forall p \geq \overline{p}$ and

$$\left.\frac{dW(q_1(p), q_2(p) = 0)}{dp}\right|_{p \geq \overline{p}} = [(1+\alpha)B_{q_1} - C_{q_1}](\partial q_1/\partial p) \gtrless 0. \quad (11)$$

[3] In the Appendix we show that the SOC is satisfied and the problem is well behaved.

The point is that when quality 2 is zero and price is above $\overline{p}$, then a marginal increase in price can be welfare improving (reducing) if the marginal benefit from quality 1 is larger (smaller) than the marginal cost.

Define p^{sb} as the second best solution price. We define this price *second best* because one dimension of quality is not contractible. In the next section 3, we derive the optimal price under the *first best*, i.e. when the two dimensions of quality are contractible.

The following proposition establishes conditions that ensure $p^{sb} = p^*$. As we show below, p^{sb} can be different from p^* defined above.

Proposition 1 *Suppose that: (i)* $B_{q_1}(p = 0) \geq B_{q_2}(p = 0)$*; (ii)* $(-\alpha B_{q_2q_2} + C_{q_2q_2}) > (C_{q_1q_2} - \alpha B_{q_1q_2}) > 0$*; (iii)* $dW(q_1(p), q_2(p) = 0)/dp|_{p=\overline{p}} < 0$*. Then,* $dW(p = 0)/dp > 0$ *and the optimal price is below the marginal benefit of quality 1:*

$$p^{sb} = p^* = B_{q_1} - B_{q_2} \left(C_{q_1q_2} - \alpha B_{q_1q_2}\right) / \left(-\alpha B_{q_2q_2} + C_{q_2q_2}\right) < B_{q_1}. \tag{12}$$

The incentive scheme is low powered.

Proof. Appendix. ■

Suppose that conditions (i-iii) in Proposition 1 hold. Then the price is positive (condition i), the tasks are substitutes in the patients' benefit and provider's cost (condition ii) and the sponsor prefers that both dimensions of quality are provided (condition iii). This case resembles situations where both dimensions of quality are important for the sponsor but each dimensions of quality has a negative impact on the cost and benefits of the other quality dimension. In these cases, the price is positive, but *low-powered* since a too high price will crowd-out valuable quality on the non-verifiable task.

The conditions i-iii) in Proposition 1 are sufficient but not necessary for $p^{sb} = p^*$. A necessary but not sufficient condition for Proposition 1 to hold is: $B_{q_1} \left(-\alpha B_{q_2q_2} + C_{q_2q_2}\right) > B_{q_2} \left(C_{q_1q_2} - \alpha B_{q_1q_2}\right)$. This condition guarantees that $dW/dp(p = 0) > 0$ and that $p^* > 0$, but it does not establish whether p^* generates maximum welfare. Whether it does it depends of how the welfare function varies with p for $p > \overline{p}$. If $dW/dp|_{p=\overline{p}} > 0$, then p^*

might not be a maximum. We come back to this point below.

Figure 1.a and 1.b show the solution. Figure 1.b illustrates that condition (iii) in proposition 1 is a sufficient but not necessary condition for $p^{sb} = p^*$.[4] Figure 1.b shows the case where $dW/dp|_{p=\overline{p}} > 0$ but still $p^{sb} = p^*$. In Figure 1.b, the price $\widetilde{p}$ denotes the price such that $\widetilde{p} =: \left.\frac{dW(q_1(p),q_2(p)=0)}{dp}\right|_{p \geq \overline{p}} = 0$.

[Figure 1.a and 1.b]

If the optimal price $p^{sb} = p^*$ is positive, then the FOC can be rewritten as:

$$[(1+\alpha)B_{q_1} - C_{q_1}]\,(\partial q_1/\partial p) = [(1+\alpha)B_{q_2} - C_{q_2}]\,(-\partial q_2/\partial p) \tag{13}$$

The optimal price is such that the marginal welfare gain from an increase in quality dimension 1 is equal to the marginal welfare loss from a reduction in quality dimension 2.

The optimality condition of the price $p^{sb} = p^*$ can be also re-written in terms of elasticities

$$\epsilon_{q_1}^{W}\epsilon_{p}^{q_1} = \epsilon_{q_2}^{W}(-\epsilon_{p}^{q_2}) \tag{14}$$

where $\epsilon_{q_i}^{W} = \partial W/\partial q_i(q_i/W)$ denotes the elasticity of welfare with respect to quality dimension i and $\epsilon_{p}^{q_i} = \partial q_i/\partial p(p/q_i)$ the elasticity of quality i with respect to price. The optimal price is such that the product of the elasticity of welfare with respect to quality and the elasticity of quality with respect to price are equated.[5]

In some cases the purchaser will set the optimal price equal to zero. Intuitively, these are the cases where quality on dimension 2 is relatively more important for the sponsor, and when a positive price shifts the provider's choice of quality production towards the first task. The following proposition provides a sufficient condition for having no incentive scheme, ie for setting $p^{sb} = 0$.

[4] The cost and benefit functions are assumed to be quadratic. See section 4 for details. In Figure 1.a and 1.b we assume $a_1 = a_2 = b_1 = b_2 = c_1 = c_2 = 1$. For Figure 1.a we set $\alpha = 0.5$ and $m = 0.5$. For Figure 1.b we set $\alpha = 0.25$ and $m = 0.5$.

[5] From $W_{q_1}\partial q_1/\partial p = W_{q_2}\,(-\partial q_2/\partial p)$, we obtain $W_{q_1}\frac{q_1}{W}\partial q_1/\partial p\frac{p}{q_1} = W_{q_2}\frac{q_2}{W}\,(-\partial q_2/\partial p)\,\frac{p}{q_2}$.

Proposition 2 *Suppose that: (i) at* $p = 0$, $B_{q_1}(-\alpha B_{q_2 q_2} + C_{q_2 q_2}) < B_{q_2}(C_{q_1 q_2} - \alpha B_{q_1 q_2})$ *where* $(C_{q_1 q_2} - \alpha B_{q_1 q_2}) > 0$*; (ii)* $dW(q_1(p), q_2(p) = 0)/dp|_{p=\overline{p}} < 0$*. Then,* $dW(p = 0)/dp < 0$ *and* $p^{sb} = 0$*. The incentive scheme breaks down.*

Proof. Appendix. ■

Figure 2.a and 2.b illustrate the solution.[6] Intuitively, since the marginal benefit of quality dimension 1 is small compared to quality dimension 2, introducing a price reduces welfare. Condition (ii) in proposition 2 is sufficient. As figure 2.b shows, even if $dW/dp|_{p=\overline{p}} > 0$, it may still be optimal to set $p^{sb} = 0$.

[Figure 2.a and 2.b]

The following two propositions establish conditions for having *high-powered* incentive schemes.

Proposition 3 *Suppose that: (i)* $dW(q_1(p), q_2(p) = 0)/dp|_{p=\overline{p}} > 0$*; (ii)* $W(\widetilde{p}) > W(p^*)$ *or* $W(\widetilde{p}) > W(p = 0)$*. Then,* $p^{sb} = \widetilde{p} = B_{q_1}$*. The incentive scheme is high powered.*

Proof. Appendix. ■

Assumption i) ensures that welfare increases for $p > \overline{p}$ up to price $p = \widetilde{p}$. In words, when quality 2 is driven to zero, a marginal increase in price p is such that the marginal benefit from quality 1 is bigger than its marginal cost. This might be the case when the level of altruism is sufficiently low, so that quality 2 quickly drops to zero when price increases. Condition ii) guarantees that $p = \widetilde{p}$ is the global maximum.

Figures 3.a-3.c show three possible scenarios. In Figure 3.a we have $dW(p = 0)/dp < 0$. In this case an increase in the price reduces welfare for low p because it reduces a lot quality 2. However, reached price $p = \overline{p}$, the level of quality 2 is zero and therefore given assumption i) welfare increases after that up to price $p = \widetilde{p}$. Condition (ii) guarantees that $p = \widetilde{p}$ is the global maximum. In Figure 3.b we have $dW(p = 0)/dp > 0$. There is a local

[6] In Figure 2.a and 2.b we assume $a_1 = b_1 = c_1 = c_2 = 1$, $a_2 = 2$, $b_2 = 0$ and $m = 0.5$. For Figure 2.a we set $\alpha = 0.2$, while $\alpha = 0.1$ for Figure 2.b.

maximum at $p = p^*$. Again reached price $p = \overline{p}$, the level of quality 2 is zero and therefore given our assumption in (i) welfare increases after that up to price $p = \widetilde{p}$. Condition (ii) guarantees that $p = \widetilde{p}$ is the global maximum. In Figure 3.a and 3.b it is always the case that $p^* < \overline{p} < \widetilde{p}$. Figure 3.c provides an example where $\overline{p} < p^* < \widetilde{p}$.[7]

[Figure 3.a, 3.b, and 3.c]

Finally, our next proposition states that the incentive scheme is always high-powered when the two quality dimensions are complements in the provider's cost or patient's benefit function, i.e. when $C_{q_1q_2} - \alpha B_{q_1q_2} < 0$.

Proposition 4 *Suppose that: (i) the two quality dimensions are complements:* $C_{q_1q_2} - \alpha B_{q_1q_2} < 0$. *Then,* $p^{sb} = p^* = B_{q_1} + B_{q_2}\left(-C_{q_1q_2} + \alpha B_{q_1q_2}\right) / \left(-\alpha B_{q_2q_2} + C_{q_2q_2}\right) > B_{q_1}$. *The incentive scheme is high powered.*

Proof. Appendix. ■

The optimal price is set above the marginal benefit of quality dimension 1. In this case we do not need to worry about the constraint $q_2 \geq 0$, because it is always satisfied with strict inequality. This is because $dq_2/dp > 0$. Since an increase in price increases not only quality 1 but also quality 2, then it is optimal to increase the price at a level where the marginal benefit of quality 1 is above its marginal cost ($(1+\alpha)B_{q_1} > C_{q_1}$). The optimal price is such that

$$[(1+\alpha)B_{q_2} - C_{q_2}]\left(\partial q_2/\partial p\right) = -[(1+\alpha)B_{q_1} - C_{q_1}]\left(\partial q_1/\partial p\right). \tag{15}$$

In equilibrium the marginal welfare gain from an increase in quality 2 is equal to the marginal welfare loss from an increase in quality 1.

[7] In Figure 3.a -3.c we assume $a_1 = b_1 = c_1 = c_2 = 1$, and $m = 0.5$. For Figure 3.a we set $a_2 = 2$, $b_2 = 0$ $\alpha = 0.05$, for Figure 3.b $a_2 = b_2 = 1$, $\alpha = 0.2$, and in Figure 3.c. $a_2 = b_2 = 1$ $\alpha = 0.1$.

3 Comparison with first best

In this section we first define the first best solution and then compare the results obtained in Propositions 1-4, which we refer to as the second best solution, with the first best solution.

3.1 First best

We define the first best solution a setting where the purchaser can observe both quality dimensions and maximize over the quality levels directly. This is equivalent to set two different prices p_1 and p_2 for respectively q_1 and q_2. The purchaser maximises the difference between benefit and transfers

$$\max_{p_1 \geq 0, p_2 \geq 0} \quad W = (1+\alpha)B(q_1(p_1), q_2(p_2)) - C(q_1(p_1), q_2(p_2)) \tag{16}$$

subject to the provider's participation constraint and the incentive-compatibility (IC) constraints. (The IC-constraints follow from the provider's first order conditions).

$$\begin{gathered} T + pq_1 + p_2 q_2 + \alpha B(q_1, q_2) - C(q_1, q_2) \geq 0 \\ p_1 + \alpha B_{q_1}(q_1, q_2) - C_{q_1}(q_1, q_2) \leq 0, \quad q_1 \geq 0 \\ p_2 + \alpha B_{q_2}(q_1, q_2) - C_{q_2}(q_1, q_2) \leq 0, \quad q_2 \geq 0. \end{gathered} \tag{17}$$

The FOCs with respect to price are:

$$\frac{dW(q_1(p_1), q_2(p_2))}{dp_1} = \left[(1+\alpha)B_{q_1} - C_{q_1}\right](\partial q_1/\partial p_1) = 0$$

$$\frac{dW(q_1(p_1), q_2(p_2))}{dp_2} = \left[(1+\alpha)B_{q_2} - C_{q_2}\right](\partial q_2/\partial p_2) = 0 \tag{18}$$

Using the FOCs for the provider (the ICs) we obtain

$$p_i^{fb} = B_{q_i}\left(q_1^{fb}, q_2^{fb}\right), \qquad i = 1, 2 \tag{19}$$

$$q_i^{fb}\colon\ (1+\alpha)B_{q_i} = C_{q_i}, \qquad i = 1, 2. \tag{20}$$

Hence, the price of each quality dimension is set equal to the marginal benefit this dimension generates. Furthermore, marginal costs along each dimension of quality are equal to the marginal benefit from quality plus the the altruistic component.

3.2 Comparison of first best and second best

We start by comparing prices of quality dimension 1. However since both the marginal benefit and the marginal cost of quality 1 depends on the level of quality 2 we are not able to compare prices and quality levels without making further assumptions. To compare prices we assume that marginal benefit of quality dimension 1 is constant, and that marginal benefit of quality 1 is independent of quality 2. The following corollary compares solutions.[8]

Corollary 1 *Suppose* $B_{q_1q_1} = B_{q_1q_2} = 0$. *i) If the conditions in Proposition 1 hold then* $p_1^{fb} > p^{sb}$. *ii) If the conditions in Proposition 3 hold then* $p_1^{fb} = p^{sb}$. *iii) Suppose the two quality dimensions are complements in the provider's cost function so* $C_{q_1q_2} < 0$, *then* $p_1^{fb} < p^{sb}$.

Proof. Appendix. ■

The Corollary shows that the second best price coincides with first best price only when quality 2 is zero in the second best. However, the real allocations, i.e. the choice of quality differs in first- and second best also in this case. This follows since $q_2^{fb} > q_2^{sb} = 0$. This implies that welfare will differ in first- and second best. We now turn to comparing the level of quality under the two settings.

[8] Obvioulsy, when the price in second best is zero (Proposition 2), the first best price is higher.

Comparing the levels of quality is not straightforward. The problem is that even if we can rank the prices for quality 1, marginal costs of quality 1 depends on the level of quality 2. To compare the levels of quality we impose the following restrictions. First, we assume that marginal benefit of quality 1 is constant and that the benefit function is symmetric, $B_{q_1q_1} = B_{q_2q_2} = B_{q_1q_2} = 0$, so that $B_{q_1} = B_{q_2} = B$. Second, let the cost function be symmetric, $C_{q_1q_1} = C_{q_2q_2}$. (For simplicity) let the third-order derivatives on costs be zero so $C_{q_iq_i} = c > C_{q_iq_j} = m > 0$, where the inequality follows from the second-order conditions of the provider's maximization problem.[9]

Under these assumptions it follows from equation (19) and (20) that prices and quality for both tasks are identical in the first best. Furthermore, it follows from the provider's first-order conditions (equation (2) and (3)) that $C_{q_1^{sb}}(q_1^{sb}, q_2^{sb}) > C_{q_2^{sb}}(q_1^{sb}, q_2^{sb})$ if and only if $p^{sb} > 0$. The following lemma compares quality levels in second best.

Lemma 1 *Let i)* $B_{q_1q_2} = B_{q_1q_1} = B_{q_2q_2} = 0$, *ii)* $B_{q_1} = B_{q_2}$, *and iii)* $C_{q_1q_1} = C_{q_2q_2}$. *If* $p^{sb} > 0$, *then* $C_{q_1^{sb}}(q_1^{sb}, q_2^{sb}) > C_{q_2^{sb}}(q_1^{sb}, q_2^{sb}) \Rightarrow q_1^{sb} > q_2^{sb}$.

Proof. Appendix. ■

Hence, when costs and benefits are symmetric in the two quality dimensions, quality along the verifiable dimension is higher than on the non-verifiable quality dimension if the second best price is strictly positive.

From equations (3) and (20) we have

$$B = \frac{C_{q^{fb}}(q^{fb}, q^{fb})}{(1+\alpha)} = \frac{C_{q_2^{sb}}(q_1^{sb}, q_2^{sb})}{\alpha}. \tag{21}$$

Furthermore, it follows from equation (2) and (20) that

$$B = \frac{C_{q^{fb}}(q^{fb}, q^{fb})}{(1+\alpha)} = \frac{C_{q_1^{sb}}(q_1^{sb}, q_2^{sb})}{(1+\alpha-C)}, \tag{22}$$

[9] Note that the Hessian is $|H| \equiv \begin{vmatrix} -C_{q_1q_1} & -C_{q_1q_2} \\ -C_{q_1q_2} & -C_{q_2q_2} \end{vmatrix} = C_{q_1q_1}C_{q_2q_2} - (C_{q_1q_2})^2 > 0$ from SOC. Hence $C_{q_iq_i} > C_{q_iq_j}$.

where $C = \frac{C_{q_1 q_2}}{C_{q_1 q_1}} > 0$.We thus have (the last inequality follows from Lemma 1)

$$C_{q^{fb}}(q^{fb}, q^{fb}) > C_{q_1^{sb}}(q_1^{sb}, q_2^{sb}) > C_{q_2^{sb}}(q_1^{sb}, q_2^{sb}). \tag{23}$$

Obviously, the conditions given in (23) holds for $q^{fb} > q_1^{sb} > q_2^{sb}$. The following proposition gives an upper boundary for q_2^{sb} in the case where $q^{fb} < q_1^{sb}$.

Proposition 5 *Suppose the conditions in Lemma 1 hold and $q_1^{sb} > q_2^{sb}$. If $q^{fb} < q_1^{sb}$ then i) $q_2^{sb} < q^{fb} - \frac{c}{m}\left(q_1^{sb} - q^{fb}\right) < q^{fb}$ and ii) $q_1^{sb} + q_1^{sb} < 2q^{fb}$.*

Proof. Appendix. ■

Hence, if $q^{fb} < q_1^{sb}$, then quality along the second dimension is below the first best quality level. Furthermore, aggregate quality is lower in the second best compared with the first best. Since patients' marginal benefit on both quality dimensions is equal, patients benefits are higher in the first best.

4 Comparative statics of price

In this section we provide some comparative statics results in the case where benefit and costs are quadratic. Suppose that $B(q_1, q_2) = a_1 q_1 - (b_1/2)q_1^2 + a_2 q_2 - (b_2/2)q_2^2$ and $C(q_1, q_2) = (c_1/2)q_1^2 + (c_2/2)q_2^2 + m q_1 q_2$. By solving the provider's first order conditions (equation (2) and (3)) for the quality levels we obtain

$$\begin{aligned} q_1^*(p) &= \frac{(p + \alpha a_1)(c_2 + \alpha b_2) - m\alpha a_2}{c_1 c_2 + \alpha b_1 c_2 + \alpha b_2 c_1 + \alpha^2 b_1 b_2 - m^2} \\ q_2^*(p) &= \frac{\alpha a_2 (c_1 + \alpha b_1) - m(p + \alpha a_1)}{c_1 c_2 + \alpha b_1 c_2 + \alpha b_2 c_1 - m^2 + \alpha^2 b_1 b_2} \end{aligned} \tag{24}$$

Then, the optimal price is (follows from equation (10)):

$$p^{sb} = p^* = B_{q_1} - B_{q_2} \frac{m}{\alpha b_2 + c_2}. \tag{25}$$

First, we consider the case where marginal benefit is constant. In this case the optimal price is decreasing (increasing) in $C_{q_1q_2} = m > 0 (< 0)$. That is, it decreasing (increasing) when the two quality dimensions are substitutes (complements) in the provider's cost function. Then we show that if marginal benefits are decreasing, the optimal price can be increasing in $C_{q_1q_2} = m$ even in the case where the quality dimensions are substitutes in the provider's cost function, i.e. when $m > 0$.

4.1 Constant marginal benefit

Suppose that the marginal benefit is constant ($b_1 = b_2 = 0$). Then the optimal price is

$$p^{sb} = p^* = a_1 - m\frac{a_2}{c_2} \tag{26}$$

Notice that if the marginal benefit of quality 2 is sufficiently high and the marginal cost sufficiently low, then p^* might be negative, in which case $p^{sb} = 0$. In the following we assume that p^* is positive.

The optimal price p^{sb} is increasing in the marginal benefit of quality 1 and decreasing in the marginal benefit of quality 2. The price is decreasing in m, as expected: the more the two quality dimensions are substitutes, the smaller is the price ($\partial p/\partial m < 0$).

The higher the marginal cost of dimension 2 the higher is the price ($\partial p/\partial c_2 = a_2 m/c_2^2 > 0$). This is somewhat counter-intuitive, but follows from the fact that an increase in the marginal cost of quality 2 reduces quality 2 and thus the marginal cost of quality 1. Since marginal benefit is constant the provider's first-order condition (equation 2) is violated if the price remains constant, i.e. $\overline{p}^{sb} - \alpha a_1 > C_{q_1}(q_1, q_2 - \Delta)$, where $\Delta > 0$ is the reduction in quality 2 that follows the increase in marginal cost of quality 2. The payer thus responds to an increase in the marginal cost of quality 2, c_2, by increasing the price, which as showed below, increases the level of quality 1.

Finally, notice that price does not vary with altruism nor with the marginal cost of quality 1 (as price is equal to marginal benefit). Note that the above results also hold for small b_1 and b_2 (i.e. for $b_1 \to 0$ and $b_2 \to 0$).

Substituting the optimal price into the FOCs of the provider, we obtain

$$q_1^* = \frac{(1+\alpha)(a_1c_2 - ma_2)}{c_1c_2 - m^2};\; q_2^* = \frac{\alpha(a_2c_1 - ma_1) - \frac{m}{c_2}(a_1c_2 - ma_2)}{c_1c_2 - m^2}. \tag{27}$$

The following corollary establishes how the optimal levels of quality vary with the different parameters.

Corollary 2 *Suppose* $p^* > 0$. *(a)* $\partial q_i^*/\partial a_i > 0$ *with* $i = 1, 2$*; (b)* $\partial q_i^*/\partial a_j < 0$ *with* $j = 1, 2$ *and* $i \neq j$*; (c)* $\partial q_i^*/\partial c_i < 0$ *with* $i = 1, 2$*; (d)* $\partial q_i^*/\partial c_j > 0$ *with* $j = 1, 2$ *and* $i \neq j$*; (e)* $\partial q_i^*/\partial \alpha > 0$ *with* $i = 1, 2$*; (f)* $\partial q_i^*/\partial m \gtrless 0$ *with* $i = 1, 2$.

Proof. Appendix. ■

The effect of each parameter on quality reflects the sum of the direct effect on quality plus the indirect effect through the price (see equation (24)).

(a) Corollary 1 suggests that each quality level is increasing in its marginal benefits ($\partial q_i^*/\partial a_i > 0$, $i = 1, 2$): a higher marginal benefit from quality implies a higher price but also a stronger altruistic component for the provider. Both effects induce higher quality in equilibrium.

(b) Each quality level is decreasing in the marginal benefit of the other quality dimension ($\partial q_i^*/\partial a_j < 0$, $i = j = 1, 2$ and $i \neq j$). For example, an increase in a_1 decreases q_2^* for two reasons: for a given price, a higher marginal benefit of quality 1 decreases quality 2 but also implies a higher price which also decreases quality 2.

(c) Each quality level is also decreasing in its own marginal cost, i.e. $\partial q_i^*/\partial c_i < 0$, $i = 1, 2$. A higher marginal cost for quality 1, c_1, decreases quality 1 (direct effect) and has no effect on price (follows from equation (26)). A higher marginal cost for quality 2, c_2, decreases quality 2 (direct effect). In this case there is also an indirect effect via the price; an increase in c_2 increases the price which further decreases quality 2.

(d) Each quality level is increasing in the marginal cost of the other quality dimension. A higher marginal cost for quality 2 c_2 reduces the optimal quality 2 and therefore reduces the marginal cost of quality 1 which increases quality 1; moreover it implies a higher price which also increases quality 1. Similarly, a higher marginal cost of quality 1 c_1 reduces the optimal quality 1 and therefore reduces the marginal cost of quality 2 which increases quality 2 (there is no effect through the price).

(e) Higher altruism increases the marginal benefit of quality and therefore increases quality (direct effect) and has no effect on the price.

(f) The effects of an increase in m is symmetric so we only consider the effect on q_1. A higher m implies a more negative spillover effect of a high level of q_1 on the marginal cost of providing quality 2. This effect tends to reduce quality 1. However, a higher m also implies a tendency to reduce q_2 : this effect tends to increase q_1. Which effect dominates depends on the relative size of the marginal costs of producing q_1 and q_2, and the relative marginal benefits (a_1 and a_2). If the relative benefits favour quality dimension 1 (a_1 large relative to a_2, and large relative to marginal costs) then q_1 tends to increase with m, while q_2 tends to decrease with m. A similar result occurs if the marginal cost of providing q_1 is relatively small to the marginal costs of providing q_2 (and the difference in marginal benefits is small).

4.2 Decreasing marginal benefit

We now consider the case with decreasing marginal benefit. The point we want to make is that the effects of an increase in m are quite complicated and most often indeterminate when the marginal benefit is decreasing. It might indeed be the case that the price is actually increasing in m. This happens when marginal benefits decreases sufficiently fast.

The optimal price is now given by:

$$p^{sb} = a_1 - b_1 q_1(p^{sb}) - \left[a_2 - b_2 q_2(p^{sb})\right] \frac{m}{\alpha b_2 + c_2} \tag{28}$$

Consider the effect of an increase in $m > 0$. Totally differentiating we obtain:

$$\begin{aligned} & \left[1 + b_1 \frac{\partial q_1(p^{sb})}{\partial p} - \frac{b_2 m}{\alpha b_2 + c_2} \frac{\partial q_2(p^{sb})}{\partial p}\right] dp^{sb} \\ & + \left[\frac{a_2 - b_2 q_2(p^{sb})}{\alpha b_2 + c_2} + b_1 \frac{\partial q_1}{\partial m} - \frac{b_2 m}{\alpha b_2 + c_2} \frac{\partial q_2}{\partial m}\right] dm \\ = & \ 0 \end{aligned} \tag{29}$$

Now, since $\partial q_1(p^{sb})/\partial p > 0$ and $\partial q_2(p^{sb})/\partial p < 0$, then

$$sign \frac{dp^{sb}}{dm} \Longleftrightarrow sign \left[-\frac{a_2 - b_2 q_2(p^{sb})}{\alpha b_2 + c_2} - b_1 \frac{\partial q_1}{\partial m} + \frac{b_2 m}{\alpha b_2 + c_2} \frac{\partial q_2}{\partial m}\right]. \tag{30}$$

If $b_1 = b_2 = 0$ we obtain as a special case the previous result, so that $dp^{sb}/dm < 0$.

To show that the optimal price can increase in $m > 0$ when the marginal benefit decreases sufficiently fast, let $\alpha = 0.8$, $a_1 = a_2 = b_1 = b_2 = 2$, and let $c_1 = c_2 = 1$. The next figure shows that $dp^{sb}/dm > 0$ for $m > 0$.[10]

[Figure 4]

5 Conclusions

Purchasers make increased use of pay-for-performance incentive schemes in the attempt of fostering quality in the health care sector. However, inevitably some dimensions of quality remain not contractible. Existing incentive schemes have been criticised on the ground that paying for quality will increase quality in the dimensions that are contractible but will reduce quality for the dimensions that are not contractible. This criticism then raises the question whether such incentive schemes should be introduced, and if introduced how powered should the incentive schemes be.

We show that in some cases *low powered* incentive schemes are optimal. Introducing the scheme is useful in increasing welfare when the quality that is contractible is relatively important. However, this needs to be traded-off with the reductions in the quality dimen-

[10] With these parameter values $q_1 > 0, q_2 > 0$, and the SOC is fullfiled.

sion that is not contractible. In other cases it is optimal not to introduce an incentive scheme. This is likely to be the case when the quality dimensions that are not contractible are relatively more important.

Finally, there are some cases where the optimal incentive scheme is *high powered*. This arises in two circumstances. First, if the quality dimension that is not contractible falls to zero quickly with price (due for example to low altruism), the benefit from increasing the quality in the dimensions that are contractible can be quite large. Second, if the different quality dimensions face some complementarity, then providers become better at providing also the dimensions of quality that are not contractible, when induced to increase the quality dimensions that are contractible.

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7 Appendix

In this appendix we provide details regarding some of the calculations in this paper.

The second order conditions (SOCs) of the provider's problem are:

$$U_{q_1q_1} = \alpha B_{q_1q_1} - C_{q_1q_1} < 0, \quad U_{q_2q_2} = \alpha B_{q_2q_2} - C_{q_2q_2} < 0 \tag{31}$$

$$U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2 = \left(\alpha B_{q_1q_1} - C_{q_1q_1}\right)\left(\alpha B_{q_2q_2} - C_{q_2q_2}\right) - \left(\alpha B_{q_1q_2} - C_{q_1q_2}\right)^2 > 0 \tag{32}$$

A sufficient but not necessary condition for $U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2 > 0$ to be satisfied is $C_{q_1q_1}C_{q_2q_2} > C_{q_1q_2}^2$ and $B_{q_1q_1}B_{q_2q_2} > B_{q_1q_2}^2$.

To find the effects $\partial q_i/\partial p$, $i = 1, 2$ we use Cramer's rule. Define

$$F^1\left(q_1, q_2; p, \alpha\right) = p + \alpha B_{q_1}(q_1, q_2) - C_{q_1}(q_1, q_2) = 0$$

$$F^2\left(q_1, q_2; p, \alpha\right) = \alpha B_{q_2}(q_1, q_2) - C_{q_2}(q_1, q_2) = 0.$$

The Jacobian determinant is

$$|J| \equiv \begin{vmatrix} \alpha B_{q_1q_1} - C_{q_1q_1} & \alpha B_{q_1q_2} - C_{q_1q_2} \\ \alpha B_{q_1q_2} - C_{q_1q_2} & \alpha B_{q_2q_2} - C_{q_2q_2} \end{vmatrix} = U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2 > 0,$$

where the last inequality follows from the assumption that the SOC is satisfied.

Since $-\partial F^1/\partial p = -1$, and $-\partial F^2/\partial p = 0$ we obtain

$$\frac{\partial q_1}{\partial p} = \frac{\begin{vmatrix} -1 & \alpha B_{q_1q_2} - C_{q_1q_2} \\ 0 & \alpha B_{q_2q_2} - C_{q_2q_2} \end{vmatrix}}{|J|} = \frac{-\alpha B_{q_2q_2} + C_{q_2q_2}}{U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2} > 0,$$

$$\frac{\partial q_2}{\partial p} = \frac{\begin{vmatrix} \alpha B_{q_1q_1} - C_{q_1q_1} & -1 \\ \alpha B_{q_1q_2} - C_{q_1q_2} & 0 \end{vmatrix}}{|J|} = \frac{\alpha B_{q_1q_2} - C_{q_1q_2}}{U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2}.$$

The SOC of the sponsor's problem is satisfied and the problem is well behaved since

$$\frac{d^2W}{d^2p} = [(1+\alpha)B_{q_1q_1} - C_{q_1q_1}]\,(\partial q_1/\partial p)^2 + [(1+\alpha)B_{q_2q_2} - C_{q_2q_2}]\,(\partial q_2/\partial p)^2 < 0.$$

Proof of Proposition 1. $\left.\frac{dW(q_1(p),q_2(p))}{dp}\right|_{p<\overline{p}} = [(1+\alpha)B_{q_1} - C_{q_1}]\,(\partial q_1/\partial p) + [(1+\alpha)B_{q_2} - C_{q_2}]\,(\partial q_2/\partial p)$. Using the FOCs $p+\alpha B_{q_1} = C_{q_1}$; $\alpha B_{q_2}(q_1,q_2) = C_{q_2}(q_1,q_2)$, we have

$$\begin{aligned} & \left.\frac{dW(q_1(p),q_2(p))}{dp}\right|_{p<\overline{p}} \\ = \; & (B_{q_1} - p)\,(\partial q_1/\partial p) + B_{q_2}\,(\partial q_2/\partial p) \\ = \; & \frac{(B_{q_1} - p)\,(-\alpha B_{q_2q_2} + C_{q_2q_2}) + B_{q_2}\,(-C_{q_1q_2} + \alpha B_{q_1q_2})}{U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2} \\ = \; & \frac{B_{q_1}\,(-\alpha B_{q_2q_2} + C_{q_2q_2}) + B_{q_2}\,(-C_{q_1q_2} + \alpha B_{q_1q_2}) - p\,(-\alpha B_{q_2q_2} + C_{q_2q_2})}{U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2} \end{aligned}$$

Therefore, at $p = 0$ the condition is positive when (i) and (ii) are satisfied. ■

Proof of Proposition 2.

$$\begin{aligned} & \left.\frac{dW(q_1(p),q_2(p))}{dp}\right|_{p<\overline{p}} \\ = \; & \frac{B_{q_1}\,(-\alpha B_{q_2q_2} + C_{q_2q_2}) + B_{q_2}\,(-C_{q_1q_2} + B_{q_1q_2}) - p\,(-\alpha B_{q_2q_2} + C_{q_2q_2})}{U_{q_1q_1}U_{q_2q_2} - U_{q_2q_1}^2} \end{aligned}$$

At $p = 0$ the condition is $\left.\frac{dW(q_1(p),q_2(p))}{dp}\right|_{p<\overline{p}} = \frac{B_{q_1}\left(-\alpha B_{q_2q_2}+C_{q_2q_2}\right)+B_{q_2}\left(-C_{q_1q_2}+B_{q_1q_2}\right)}{U_{q_1q_1}U_{q_2q_2}-U_{q_2q_1}^2}$. Condition (i) is only necessary. ■

Proof of Proposition 3. $\left.\frac{dW(q_1(p),q_2(p)=0)}{dp}\right|_{p\geq\overline{p}} = [(1+\alpha)B_{q_1} - C_{q_1}]\,(\partial q_1/\partial p)$. Using the FOC $p+\alpha B_{q_1} = C_{q_1}$, then $\left.\frac{dW(q_1(p),q_2(p)=0)}{dp}\right|_{p\geq\overline{p}} = (B_{q_1} - p)\,(\partial q_1/\partial p) = 0$, which implies $\widetilde{p} = B_{q_1}$. ■

Proof of Proposition 4. In this case the quality dimension 2 is always strictly positive, $q_2 > 0$ and therefore $p = \widetilde{p}$ cannot be in equilibrium. The solution is given by p^*. ■

Proof of Corollary 1. To prove the first statement, recall that $p_1^{fb} = B_{q_1}$, $C_{q_1q_2} < 0$, and $p^{sb} = B_{q_1} + B_{q_2}\,(-C_{q_1q_2})\,/\,(-\alpha B_{q_2q_2} + C_{q_2q_2})$. Note that since $B_{q_1q_1} = 0$, then B_{q_1}

is a constant. Therefore, since $B_{q_2}(-C_{q_1q_2})/(-\alpha B_{q_2q_2}+C_{q_2q_2})>0$ then $p_1^{fb}>p^{sb}$. The second statement follows since $p^{sb}=\widetilde{p}=B_{q_1}$ when the conditions in Proposition 3 holds. The proof of the last statement follows along similar lines as the first statement but now $C_{q_1q_2}<0$. ■

Proof of Lemma 1. Suppose $q_1=q_2=q$. Then $C_{q_1}=C_{q_2}=K$ (by symmetry of the cost function). Let $\Delta>0$. Then (starting in a symmetric situation)

$$K_1 \equiv C_{q_1}(q+\Delta,q)\approx K+c\Delta>K+m\Delta\approx C_{q_2}(q+\Delta,q)\equiv K_2.$$

Now, let $q_1=q+\Delta>q_2$ (starting in an asymmetric situation).

$$C_{q_1}(q_1+\Delta,q_2)\approx K_1+c\Delta>K_2+m\Delta\approx C_{q_2}(q_1+\Delta,q_2).$$

Since Δ can be chosen arbitrarily small the approximation should hold. Furthermore, symmetry ensures that $C_{q_1}\leq C_{q_2} \iff q_2\geq q_1$. ■

Proof of Proposition 5. We have

$$\begin{aligned}
C_{q^{fb}}(q^{fb},q^{fb}) &= (c+m)\,q^{fb}>cq_1^{sb}+mq_2^{sb}=C_{q_1^{sb}}(q_1^{sb},q_2^{sb})\\
\frac{c}{m}q^{fb}+q^{fb} &> \frac{c}{m}q_1^{sb}+q_2^{sb}\\
q^{fb}-\frac{c}{m}\left(q_1^{sb}-q^{fb}\right) &> q_2^{sb}.
\end{aligned}$$

Hence, if $q^{fb}<q_1^{sb}$, then quality along the second dimension cannot be too high.

The second statement follows since $q^{fb}-q_1^{sb}$ and $0<m<c$. Hence

$$-m\left(q^{fb}-q_2^{sb}\right)<c\left(q^{fb}-q_1^{sb}\right)<0$$

This last equation is fulfilled if and only if $-\left(q^{fb}-q_2^{sb}\right)<\left(q^{fb}-q_1^{sb}\right)$, or $q_1^{sb}+q_2^{sb}<2q^{fb}$. ■

Proof of Corollary 2.

$$
\begin{aligned}
\text{(a)}\ \frac{\partial q_1^*}{\partial a_1} &= \frac{(1+\alpha)\,c_2}{c_1c_2-m^2} > 0, \quad \frac{\partial q_2^*}{\partial a_2} = \frac{\alpha c_1 + m^2/c_2}{c_1c_2-m^2} > 0 \\
\text{(b)}\ \frac{\partial q_1^*}{\partial a_2} &= -\frac{(1+\alpha)\,m}{c_1c_2-m^2} < 0, \quad \frac{\partial q_2^*}{\partial a_1} = -\frac{m\,(1+\alpha)}{c_1c_2-m^2} < 0 \\
\text{(c)}\frac{\partial\,(q_1^*)}{\partial c_1} &= -\frac{(1+\alpha)\,(a_1c_2-ma_2)\,c_2}{\left(c_1c_2-m^2\right)^2} < 0 \\
\frac{\partial\,(q_2^*)}{\partial c_2} &= -\frac{m^2a_2}{c_2^2\,(c_1c_2-m^2)} - c_1\frac{\left(\alpha\,(a_2c_1-ma_1) - \frac{m}{c_2}\,(a_1c_2-ma_2)\right)}{\left(c_1c_2-m^2\right)^2} \\
&= -\frac{1}{(c_1c_2-m^2)}\left(\frac{m^2a_2}{c_2^2} + c_1q_2^*\right) < 0. \\
\text{(d)}\ \frac{\partial q_1^*}{\partial c_2} &= \frac{m\,(1+\alpha)\,(a_2c_1-ma_1)}{\left(c_1c_2-m^2\right)^2} > 0 \text{ since } (a_2c_1-ma_1) > 0 \text{ if } q_2^* > 0. \\
\frac{\partial q_2^*}{\partial c_1} &= (1+\alpha)\,m\frac{a_1c_2-ma_2}{\left(c_1c_2-m^2\right)^2} > 0 \\
\text{(e)}\ \frac{\partial q_1^*}{\partial \alpha} &= \frac{a_1c_2-ma_2}{c_1c_2-m^2} > 0, \quad \frac{\partial q_2^*}{\partial \alpha} = \frac{a_2c_1-ma_1}{c_1c_2-m^2} > 0
\end{aligned}
$$

To prove statement (f), note that

$$
\begin{aligned}
\frac{\partial q_1^*}{\partial m} &= -\frac{c_2\,(1+\alpha)\left[(a_2c_1-ma_1) - \frac{m}{c_2}(a_1c_2-ma_2)\right]}{\left(c_1c_2-m^2\right)^2}, \\
\frac{\partial q_2^*}{\partial m} &= (1+\alpha)\,\frac{m\,(a_2c_1-ma_1) - c_1\,(a_1c_2-ma_2)}{\left(c_1c_2-m^2\right)^2}.
\end{aligned}
$$

Hence

$$
\begin{aligned}
\operatorname{sign}\left(\frac{\partial q_1^*}{\partial m}\right) &= \operatorname{sign}\left[\frac{m}{c_2}(a_1c_2-ma_2) - (a_2c_1-ma_1)\right] \\
&= \operatorname{sign}\left[2ma_1 - a_2\left(\frac{m^2}{c_2} + c_1\right)\right], \\
\operatorname{sign}\left(\frac{\partial q_2^*}{\partial m}\right) &= \operatorname{sign}\left[m\,(a_2c_1-ma_1) - c_1\,(a_1c_2-ma_2)\right]. \\
&= \operatorname{sign}\left[2ma_2 - a_1\left(\frac{m^2}{c_1} + c_2\right)\right].
\end{aligned}
$$

Hence, $\frac{\partial q_i^*}{\partial m} \gtreqless 0$ depending on the parameter values. ■

Case 1: Low-powered incentive scheme. $p^{sb} = p^*$

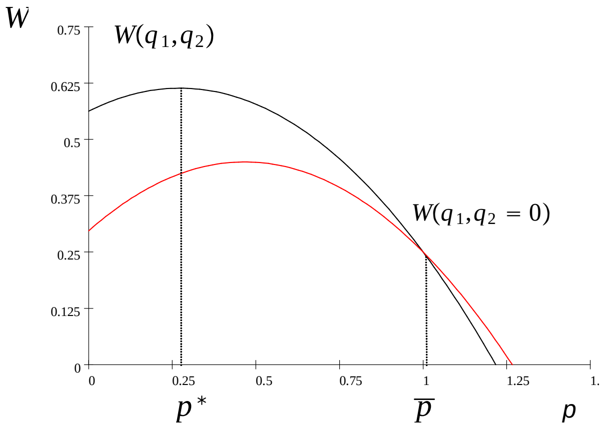


Figure 1a

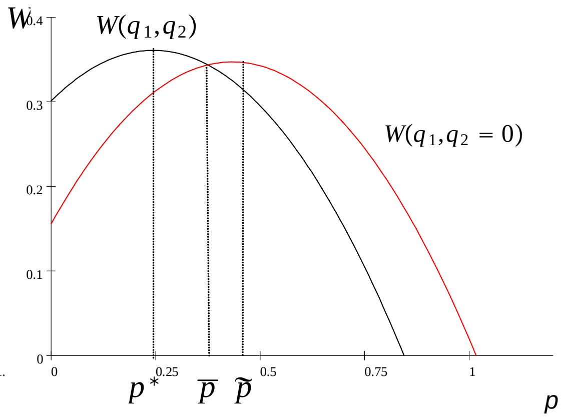


Figure 1b

Case 2: Incentive scheme breaks down. $p^{sb} = 0$

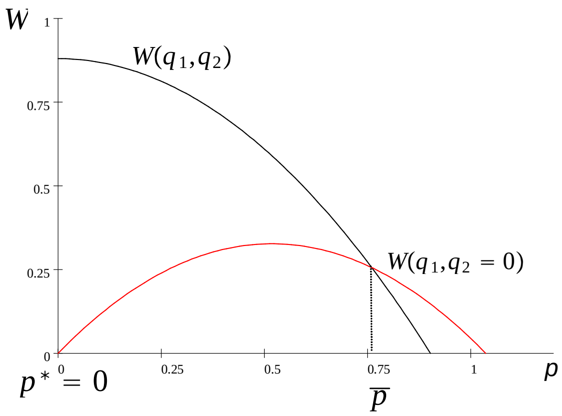


Figure 2a

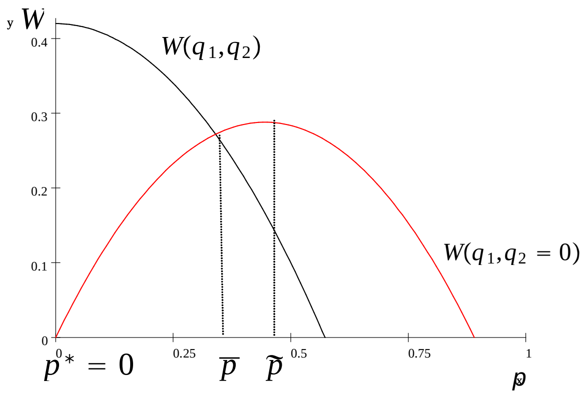
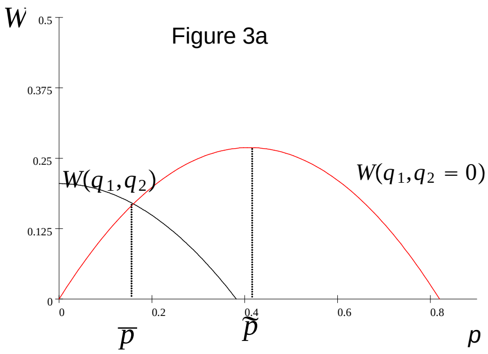
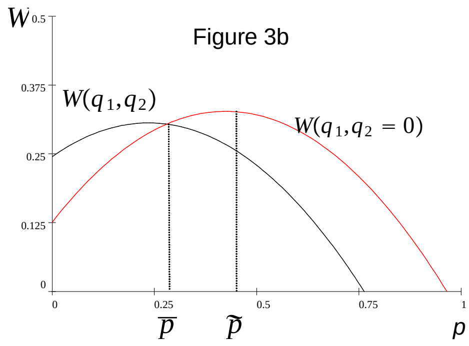


Figure 2b

Case 3: Incentive scheme is high powered





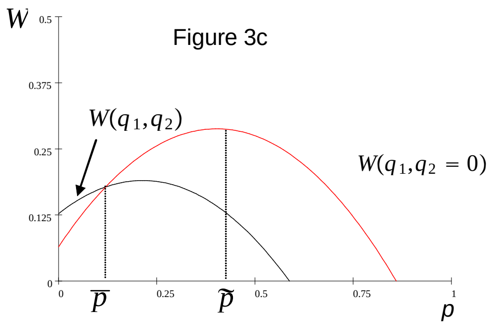


Figure 4. Optimal price as a function of *m*